Assignment-2

09/06/21

Questions on Differentiation:

① A particle moves along the curve $x = t^3+1, y = t^2,$ $z = 2t+5$, where 't' is the time. Find the components of its velocity and acceleration at time $t=1$ in the direction $2\hat{i} + 3\hat{j} + 6\hat{k}$.

$\left[\frac{24}{7}, \frac{18}{7}\right]$

② The position vector of a particle at time 't' is $\vec{r} = \cos(t-1)\hat{i} + \sinh(t-1)\hat{j} + \alpha t^3 \hat{k}$. Find the condition imposed on α by requiring that at time $t=1$, the acceleration is normal to the position vector.

$\left[\alpha = \pm \frac{1}{\sqrt{6}}\right]$

③ At any point of the curve
$x = 3\cos t,\ y = 3\sin t,\ z = 4t$, find
(i) Tangent vector. (ii) Unit tangent vector.

④ Let $f(x,y,z)$ and $g(x,y,z)$ be two scalar functions. Find an expression for $\nabla^2(fg)$ in terms of $\nabla^2 f$, $\nabla^2 g$, ∇f and ∇g.

$\left[f(\nabla^2 g) + g(\nabla^2 f) + 2(\nabla f)(\nabla g)\right]$

⑤ Evaluate grad ϕ if $\phi = \log(x^2+y^2+z^2)$

[Ans: $\dfrac{2(x\hat{i} + y\hat{j} + z\hat{k})}{x^2+y^2+z^2}$

⑥ A vector $\bar{r}$ is defined by $\bar{r} = ix + jy + kz$. If $|\bar{r}| = r$ then show that the vector $r^n \bar{r}$ is irrotational.

$[\nabla \times \bar{F} = 0]$

⑦ Show that $\vec{F} = (y^2 + 2xz^2)\hat{i} + (2xy - z)\hat{j} + (2x^2 z - y + 2z)\hat{k}$ is irrotational and hence find its scalar potential.

$[\nabla \times F = 0 \Rightarrow F = \nabla(u)$
$u = xy^2 - yz + x^2 z^2 + z^2 + c]$

Questions on Integration

① If a force $\bar{F} = 2x^2 y\hat{i} + 3xy\hat{j}$ displaces a particle in the xy-plane from $(0,0)$ to $(1,4)$ along a curve $y = 4x^2$. Find the work done.

$[\frac{104}{5}$ Ans$]$

② If $F = \nabla\phi$ show that the work done in moving a particle in the force field F from $A(x_1, y_1, z_1)$ to $B(x_2, y_2, z_2)$ is independent of the path joining the two points.

③ Using Green's theorem evaluate $\int_c (x^2 y\,dx + x^2 dy)$ where c is the boundary described counter clockwise of the triangle with vertices $(0,0), (1,0), (1,1)$

(4) Using Green's theorem, evaluate

$$\int_C (x^2+xy)\,dx + (x^2+y^2)\,dy$$ where C is the square formed by the lines $y = \pm 1$, $x = \pm 1$